

Testing the Realistic Seesaw Model with Two Heavy Majorana Neutrinos at the CERN Large Hadron Collider

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Abstract

In the conventional type-(I+II) seesaw model, the effective mass matrix of three known light neutrinos is given by $M_\nu = M_L - M_D M_R^{-1} M_D^T$ in the leading-order approximation. We propose an intriguing scenario, in which the structural cancellation condition $M_D M_R^{-1} M_D^T = \mathbf{0}$ is guaranteed by the $A_4 \times Z_2$ flavor symmetry. As a consequence, neutrino masses are mainly generated by the Higgs triplet $M_\nu = M_L$, while the neutrino mixing matrix is non-unitary and takes on the nearly tri-bimaximal pattern. A discriminating feature of this scenario from the pure type-II seesaw model is that the lepton-number-violating signatures induced by the heavy Majorana neutrinos can be discovered at the CERN Large Hadron Collider. We calculate the total cross section of the same-sign dilepton events $pp \rightarrow l_\alpha^\pm N_i \rightarrow l_\alpha^\pm l_\beta^\pm jj$ (for $i = 1, 2$ and $\alpha, \beta = e, \mu, \tau$), and emphasize the significant interference of the contributions from two different heavy Majorana neutrinos. The background from the standard model and the kinematic cuts used to reduce it have been considered. The possible way to distinguish between the signals from heavy Majorana neutrinos and those from doubly-charged Higgs bosons is briefly discussed.

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I. INTRODUCTION

Recent neutrino oscillation experiments have provided us with very convincing evidence that neutrinos are indeed massive and lepton flavors do mix [1]. This great discovery implies that the Standard Model (SM) of elementary particle physics is actually incomplete. In order to accommodate tiny neutrino masses, one can naturally extend the SM by introducing three right-handed neutrinos, which are singlets under the $SU(2)_L \times U(1)_Y$ gauge group. In this case, the gauge invariance allows right-handed singlet neutrinos to have a Majorana mass M_R , whose scale is not subject to the gauge symmetry breaking and then can be much larger than the electroweak scale $\Lambda_{EW} \sim 10^2$ GeV, e.g. $\mathcal{O}(M_R) \sim 10^{14}$ GeV $\gg \Lambda_{EW}$. Therefore, the effective mass matrix for three known light neutrinos is given by $M_\nu = -M_D M_R^{-1} M_D^T$ in the leading-order approximation, with M_D being the Dirac neutrino mass matrix and $\mathcal{O}(M_D) \sim \Lambda_{EW}$. The smallness of neutrino masses can then be ascribed to the largeness of heavy Majorana neutrino masses. This is the so-called canonical seesaw mechanism [2], which offers an elegant way to explain the tiny neutrino masses.

However, the canonical seesaw model faces two serious difficulties. First, heavy Majorana neutrinos are too heavy and their interactions are too weak for them to be generated in collider experiments, especially in the forthcoming CERN Large Hadron Collider (LHC). Thus the canonical seesaw model may lose the experimental testability. Second, the ultrahigh energy scale characterized by the masses of right-handed neutrinos will cause the seesaw hierarchy problem unless $\mathcal{O}(M_R) \lesssim 10^7$ GeV [3]. One way out of these tight corners is just to lower the seesaw scale down to TeV, but make the charged-current interactions of heavy Majorana neutrinos sizable. This possibility can be realized if and only if the structural cancellation condition $M_D M_R^{-1} M_D^T = \mathbf{0}$ is fulfilled [4, 5]. The clear signatures of heavy Majorana neutrinos at the LHC are then the same-sign dilepton events $pp \rightarrow l_\alpha^\pm l_\beta^\pm jj$ (for $\alpha, \beta = e, \mu, \tau$) [6]. Apart from heavy Majorana neutrinos, the tiny neutrino masses can be attributed to the Higgs triplet Δ , which couples to two lepton doublets and acquires a small vacuum expectation value $v_\Delta = \langle \Delta \rangle$ [7]. In this case, the mass matrix of light neutrinos is given by $M_\nu = M_L = Y_\Delta v_\Delta$, where Y_Δ is the triplet Yukawa coupling matrix. The more general case is the type-(I+II) seesaw model, in which both heavy Majorana neutrinos and the Higgs triplet are present and equally contribute to light neutrino masses. Consequently, we obtain $M_\nu = M_L - M_D M_R^{-1} M_D^T$. The interplay between the terms M_L and $M_D M_R^{-1} M_D^T$

in the conventional type-(I+II) seesaw model has been discussed in Ref. [8]. Different from the canonical seesaw model, the experimental testability of heavy Majorana neutrinos is preserved in this scenario if the global cancellation condition $M_L - M_D M_R^{-1} M_D^T = \mathbf{0}$ is satisfied [9]. Under this condition, both heavy Majorana neutrinos and the doubly-charged component of the Higgs triplet can be tested at the LHC via the lepton-number-violating (LNV) processes [10].

In this paper, we propose a novel type-(I+II) seesaw model, in which the contributions from heavy Majorana neutrinos to light neutrino masses are vanishing $M_D M_R^{-1} M_D^T = \mathbf{0}$. Thus the mass matrix of three known neutrinos is $M_\nu = M_L$, which is the same as in the pure type-II seesaw model [7]. However, the apparent difference of our scenario is that the neutrino mixing matrix becomes non-unitary. Furthermore, the heavy Majorana neutrinos can be discovered at the LHC as in the testable canonical seesaw model. Recently, the collider signals of three heavy Majorana neutrinos have been considered in Ref. [11] in the type-I seesaw model, which extends the previous works about only one heavy Majorana neutrino case [12]. Another purpose of the present work is to consider the collider signatures of more than one heavy Majorana neutrinos in a realistic type-(I+II) seesaw model.

The remaining part of our paper is organized as follows. In Section II, we propose an interesting type-(I+II) model with only two heavy Majorana neutrinos, in which the structural cancellation condition $M_D M_R^{-1} M_D^T = \mathbf{0}$ is achieved by imposing an $A_4 \times Z_2$ flavor symmetry. As a result of this elegant symmetry, the nearly tri-bimaximal neutrino mixing can be obtained. Section III is devoted to calculating the cross sections of the processes $pp \rightarrow l_\alpha^\pm N_i \rightarrow l_\alpha^\pm l_\beta^\pm jj$, where N_i (for $i = 1, 2$) are the heavy Majorana neutrinos. The interference effects of different heavy Majorana neutrinos are emphasized, and the possible way to distinguish between the same-sign dilepton signals from heavy Majorana neutrinos and the doubly-charged Higgs bosons is briefly discussed. Furthermore, the background from the SM have been taken into account, and the kinematic cuts are imposed to efficiently select the signal events. Finally, we give some concluding remarks in Section IV.

II. TESTABLE TYPE-(I+II) SEESAW MODELS

In order to generate tiny neutrino masses, we can extend the SM by introducing three right-handed neutrinos and a triplet scalar. The gauge-invariant Lagrangian relevant for

lepton masses can be written as

$$-\mathcal{L}_{\text{lepton}} = \overline{\ell}_L Y_l E_R H + \overline{\ell}_L Y_\nu N_R \tilde{H} + \frac{1}{2} \overline{N_R^c} M_R N_R + \frac{1}{2} \overline{\ell}_L Y_\Delta \Delta i\sigma_2 \ell_L^c + \text{h.c.} , \quad (1)$$

where ℓ_L and $\tilde{H} \equiv i\sigma_2 H^*$ are respectively the lepton and Higgs doublets, E_R and N_R are the right-handed charged-lepton and neutrino singlets, Δ is the triplet scalar in the 2×2 matrix form. After the spontaneous gauge symmetry breaking, the lepton mass terms turn out to be

$$-\mathcal{L}_m = \overline{e}_L M_l E_R + \frac{1}{2} \overline{(\nu_L \quad N_R^c)} \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} \nu_L^c \\ N_R \end{pmatrix} + \text{h.c.} , \quad (2)$$

where $M_L = Y_\Delta v_\Delta$ and $M_D = Y_\nu v/\sqrt{2}$ are the Majorana and Dirac neutrino mass terms with v and v_Δ being the vacuum expectation values of the doublet and triplet scalars, respectively. M_l and M_R are the charged-lepton and heavy right-handed Majorana neutrino mass matrices. The total 6×6 neutrino mass matrix can be diagonalized by the following unitary transformation

$$\begin{pmatrix} V & R \\ S & U \end{pmatrix}^\dagger \begin{pmatrix} M_L & M_D \\ M_D^T & M_R \end{pmatrix} \begin{pmatrix} V & R \\ S & U \end{pmatrix}^* = \begin{pmatrix} \widehat{M}_\nu & \mathbf{0} \\ \mathbf{0} & \widehat{M}_N \end{pmatrix} , \quad (3)$$

where $\widehat{M}_\nu = \text{Diag}\{m_1, m_2, m_3\}$ and $\widehat{M}_N = \text{Diag}\{M_1, M_2, M_3\}$ are the mass eigenvalues of light and heavy Majorana neutrinos, respectively. In the leading-order approximation, the effective neutrino mass matrix is determined by the seesaw formula

$$M_\nu = M_L - M_D M_R^{-1} M_D^T . \quad (4)$$

Thus the smallness of light neutrino masses are attributed to the heaviness of right-handed neutrinos and the smallness of M_L . It is obvious that the above equation reduces to the canonical seesaw formula $M_\nu = -M_D M_R^{-1} M_D^T$, if the triplet scalar is absent. In the basis where the mass eigenstates of charged leptons coincide with their flavor eigenstates, the leptonic charged-current interactions can be expressed as

$$-\mathcal{L}_{\text{CC}} = \frac{g}{\sqrt{2}} \overline{(e \quad \mu \quad \tau)}_L \gamma^\mu \left[V \begin{pmatrix} \hat{\nu}_1 \\ \hat{\nu}_2 \\ \hat{\nu}_3 \end{pmatrix}_L + R \begin{pmatrix} \hat{N}_1 \\ \hat{N}_2 \\ \hat{N}_3 \end{pmatrix}_L \right] W_\mu^- + \text{h.c.} , \quad (5)$$

where $\hat{\nu}_i$ and \hat{N}_i (for $i = 1, 2, 3$) stand for the mass eigenstates of three light and heavy Majorana neutrinos, respectively. It is the charged-current interactions that govern both the

production and detection of heavy Majorana neutrinos in the hadron collider experiments. From Eq. (5) and the unitarity condition $VV^\dagger + RR^\dagger = \mathbf{1}$, we can see that the neutrino mixing matrix V is non-unitary. Therefore, both the detection of heavy Majorana neutrinos and leptonic unitarity violation are determined by the couplings $R_{\alpha i}$ (for $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$), which represent distinct interaction strengths of heavy Majorana neutrinos with charged leptons. In the conventional type-(I+II) seesaw model, both terms on the right-hand side of Eq. (4) are comparable in magnitude and on the same order of the masses of three light neutrinos. In this case, the masses of heavy degrees of freedom are expected to be around the scale of grand unified theories, i.e. $\Lambda_{\text{GUT}} = 10^{16}$ GeV, so the conventional seesaw models can not be tested experimentally. In order that the heavy Majorana neutrinos can be produced and detected at the LHC, one should appeal to the following scenarios:

- *Scenario A* with $\mathcal{O}(M_L) \ll \mathcal{O}(M_\nu)$ and $\mathcal{O}(M_D M_R^{-1} M_D^T) \sim \mathcal{O}(M_\nu)$, but $\mathcal{O}(M_R) \sim \mathcal{O}(1 \text{ TeV})$ and $\mathcal{O}(R) \sim \mathcal{O}(M_D M_R^{-1}) \lesssim 10^{-1}$. This is similar to the canonical seesaw model, where the structural cancellation condition $M_D M_R^{-1} M_D^T \approx \mathbf{0}$ is required to render heavy Majorana neutrinos testable [4, 5].
- *Scenario B* with $\mathcal{O}(M_L) \sim \mathcal{O}(M_D M_R^{-1} M_D^T) \gg \mathcal{O}(M_\nu)$, but $\mathcal{O}(M_L - M_D M_R^{-1} M_D^T) \sim \mathcal{O}(M_\nu)$. This implies that the significant but incomplete global cancellation exists between M_L and $M_D M_R^{-1} M_D^T$ [9]. In this case, the collider signals at the LHC induced by heavy Majorana neutrinos and the doubly-charged component of the triplet scalar are correlated with each other [10].
- *Scenario C* with $\mathcal{O}(M_L) \sim \mathcal{O}(M_\nu)$ and $\mathcal{O}(M_D M_R^{-1} M_D^T) \ll \mathcal{O}(M_\nu)$. The latter condition is consistent with the structural cancellation condition $M_D M_R^{-1} M_D^T \approx \mathbf{0}$, so both heavy Majorana neutrinos and the triplet scalar can be discovered at the LHC. This interesting scenario has also been discussed in Ref. [13] in a very different context.

It has been observed that *Scenario B* may suffer from the problem of radiative instability and then serious fine-tunings [10]. This drawback can be avoided in *Scenario C*, if the heavy Majorana neutrinos are nearly degenerate in mass [4, 5].

In the following, we shall concentrate on *Scenario C* and propose an interesting model with several additional scalar fields and two heavy Majorana neutrinos, which may serve as a straightforward extension of the minimal type-(I+II) seesaw model [10, 14]. It is worth

mentioning that our discussions can easily be made applicable to the case with three heavy Majorana neutrinos, however, only two of them are sufficient for our purpose [15]. To realize *Scenario C*, we impose the $A_4 \times Z_2$ flavor symmetry on the generic Lagrangian of the Type-(I+II) seesaw model. The assignments of relevant lepton and scalar fields with respect to the symmetry group $SU(2)_L \times U(1)_Y \otimes A_4 \times Z_2$ are summarized as follows

$$\begin{aligned}
\ell_L &\sim (2, -1) \otimes (\underline{3}, 1) , & H &\sim (2, 1) \otimes (\underline{3}, 1) , \\
E_R &\sim (1, 1) \otimes (\underline{1}, 1) , & \phi &\sim (1, 0) \otimes (\underline{1}, -1) , \\
E'_R &\sim (1, -2) \otimes (\underline{1}', 1) , & \Delta &\sim (3, -2) \otimes (\underline{3}, 1) , \\
E''_R &\sim (1, -2) \otimes (\underline{1}'', 1) , & \Sigma &\sim (3, -2) \otimes (\underline{1}, 1) , \\
N_R &\sim (1, 0) \otimes (\underline{1}'', 1) , & N'_R &\sim (1, 0) \otimes (\underline{1}', -1) ,
\end{aligned} \tag{6}$$

where three scalar doublets H_i (for $i = 1, 2, 3$), four scalar triplets Σ and Δ_i (for $i = 1, 2, 3$), and one scalar singlet ϕ have been introduced. The gauge- and $A_4 \times Z_2$ -invariant Lagrangian responsible for lepton masses turns out to be

$$\begin{aligned}
-\mathcal{L}'_{\text{lepton}} &= y_1^l (\overline{\ell}_L H)_{\underline{1}} E_R + y_2^l (\overline{\ell}_L H)_{\underline{1}'} E''_R + y_3^l (\overline{\ell}_L H)_{\underline{1}''} E'_R + y_\nu (\overline{\ell}_L \tilde{H})_{\underline{1}'} N_R \\
&\quad + \frac{1}{2} y_\Sigma \overline{\ell}_L \Sigma i \sigma_2 \ell_L^c + \frac{1}{2} y_\Delta \overline{\ell}_L \Delta i \sigma_2 \ell_L^c + y_N \overline{N}_R^c N'_R \phi + \text{h.c.} .
\end{aligned} \tag{7}$$

Given the irreducible representations and multiplication rules of the A_4 group in [16, 17], one can immediately verify the $A_4 \times Z_2$ -invariance of $\mathcal{L}'_{\text{lepton}}$. After the spontaneous gauge symmetry breaking, the mass matrix of charged leptons is

$$M_l = \begin{pmatrix} y_1^l v_1 & y_3^l v_1 & y_2^l v_1 \\ y_1^l v_2 & y_3^l v_2 \omega & y_2^l v_2 \omega^2 \\ y_1^l v_3 & y_3^l v_3 \omega^2 & y_2^l v_3 \omega \end{pmatrix} ; \tag{8}$$

and the neutrino mass matrices are

$$M_L = \begin{pmatrix} y_\Sigma v_\Sigma & y_\Delta u_3 & y_\Delta u_2 \\ y_\Delta u_3 & y_\Sigma v_\Sigma & y_\Delta u_1 \\ y_\Delta u_2 & y_\Delta u_1 & y_\Sigma v_\Sigma \end{pmatrix} , \quad M_D = \begin{pmatrix} y_\nu v_1 & 0 \\ y_\nu v_2 \omega^2 & 0 \\ y_\nu v_3 \omega & 0 \end{pmatrix} , \quad M_R = \begin{pmatrix} 0 & y_N v_\phi \\ y_N v_\phi & 0 \end{pmatrix} , \tag{9}$$

where the vacuum expectation values are taken to be $v_i = \langle H_i \rangle$ (for $i = 1, 2, 3$), $u_i = \langle \Delta_i \rangle$ (for $i = 1, 2, 3$), $v_\Sigma = \langle \Sigma \rangle$ and $v_\phi = \langle \phi \rangle$, while $\omega = \exp(2i\pi/3)$ is the cubic root of +1. Provided the textures of M_D and M_R in Eq. (9), we can see that $M_D M_R^{-1} M_D^T = \mathbf{0}$ holds exactly. As a consequence, the non-zero neutrino masses mainly arises from the Type-II seesaw mechanism

$M_\nu = M_L$. As implied by Eq. (8) with the assumption that $v_1 = v_2 = v_3 \equiv v_H$, the charged-lepton mass matrix can be written as $M_l = U_l \cdot \text{Diag}\{\sqrt{3}y_1^l v_H, \sqrt{3}y_3^l v_H, \sqrt{3}y_2^l v_H\}$, which is simply diagonalized by the unitary matrix

$$U_l = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}; \quad (10)$$

Therefore, the masses of charged leptons are identified as $m_e = \sqrt{3}y_1^l v_H$, $m_\mu = \sqrt{3}y_3^l v_H$ and $m_\tau = \sqrt{3}y_2^l v_H$. Setting $u_1 = u_3 = 0$ and $u_2 \neq 0$, we can obtain the neutrino mass matrix

$$M_\nu = \begin{pmatrix} y_\Sigma v_\Sigma & 0 & y_\Delta u_2 \\ 0 & y_\Sigma v_\Sigma & 0 \\ y_\Delta u_2 & 0 & y_\Sigma v_\Sigma \end{pmatrix}, \quad (11)$$

which can be diagonalized by the $\pi/4$ -rotation in the 1-3 plane, namely

$$U_\nu = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 0 & \sqrt{2} & 0 \\ 1 & 0 & 1 \end{pmatrix}. \quad (12)$$

The mass eigenvalues of three known neutrinos are then given by $m_1 = |y_\Sigma v_\Sigma + y_\Delta u_2|$, $m_2 = |y_\Sigma v_\Sigma|$ and $m_3 = |y_\Sigma v_\Sigma - y_\Delta u_2|$, which can fit the observed values of two neutrino mass-squared differences $\Delta m_{21}^2 \equiv m_2^2 - m_1^2 = 8.0 \times 10^{-5} \text{ eV}^2$ and $|\Delta m_{32}^2| \equiv |m_3^2 - m_2^2| = 2.5 \times 10^{-3} \text{ eV}^2$ [1]. In the leading-order approximation, the non-unitary lepton flavor mixing matrix V is just the unitary matrix V_0 , which arises from the mismatch between the diagonalizations of M_l and M_ν . To be more explicit,

$$V \approx V_0 = U_l^\dagger U_\nu = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}}\omega^2 & \frac{1}{\sqrt{3}}\omega^2 & -\frac{1}{\sqrt{2}}e^{-i\pi/6} \\ -\frac{1}{\sqrt{6}}\omega & \frac{1}{\sqrt{3}}\omega & -\frac{1}{\sqrt{2}}e^{+i\pi/6} \end{pmatrix}, \quad (13)$$

which is equivalent to the tri-bimaximal mixing pattern [18] strongly favored by current neutrino oscillation experiments. From Eq. (9), we can observe that two heavy Majorana neutrinos are degenerate in mass. Two remarks are in order: (1) Because of $M_D M_R^{-1} M_D^T = \mathbf{0}$ and $M_1 = M_2$, the one-loop radiative corrections to light neutrino masses are extremely small and can be neglected [4]; (2) The slight breaking of the $A_4 \times Z_2$ symmetry may lead to a tiny mass split of heavy Majorana neutrinos, and then the resonant leptogenesis mechanism can be implemented to account for the baryon number asymmetry in the Universe [19].

To be more accurate, we can get the masses of three light and two heavy Majorana neutrinos after diagonalizing the total 5×5 neutrino mass matrix by a unitary transformation. The full parametrization of the corresponding 5×5 unitary matrix will involve 10 rotation angles θ_{ij} and 10 phase angles δ_{ij} (for $i, j = 1, 2, \dots, 5$ and $i < j$). Following Ref. [20], we may adopt the standard parametrization of the neutrino mixing matrix $V = AV_0$, where V_0 is a 3×3 unitary matrix

$$V_0 = \begin{pmatrix} c_{12}c_{13} & \hat{s}_{12}^*c_{13} & \hat{s}_{13}^* \\ -\hat{s}_{12}c_{23} - c_{12}\hat{s}_{13}\hat{s}_{23}^* & c_{12}c_{23} - \hat{s}_{12}^*\hat{s}_{13}\hat{s}_{23}^* & c_{13}\hat{s}_{23}^* \\ \hat{s}_{12}\hat{s}_{23} - c_{12}\hat{s}_{13}c_{23} & -c_{12}\hat{s}_{23} - \hat{s}_{12}^*\hat{s}_{13}c_{23} & c_{13}c_{23} \end{pmatrix} \quad (14)$$

and A is nearly an identity matrix

$$A = \mathbf{1} - \sum_{j=4}^5 \begin{pmatrix} s_{1j}^2 & 0 & 0 \\ \hat{s}_{1j}\hat{s}_{2j}^* & s_{2j}^2 & 0 \\ \hat{s}_{1j}\hat{s}_{3j}^* & \hat{s}_{2j}\hat{s}_{3j}^* & s_{3j}^2 \end{pmatrix} + \mathcal{O}(s_{ij}^4), \quad (15)$$

with $c_{ij} \equiv \cos \theta_{ij}$, $s_{ij} \equiv \sin \theta_{ij}$ and $\hat{s}_{ij} \equiv e^{i\delta_{ij}} s_{ij}$ (for $1 \leq i < j \leq 5$). It is worthwhile to note that the deviation of A from the identity matrix measures the unitary violation of neutrino mixing matrix, which is constrained to be below the percent level [21]. Therefore, it is an excellent approximation to neglect the higher-order terms $\mathcal{O}(s_{ij}^4)$ in Eq. (15). Meanwhile, the parametrization of R can be taken as

$$R = \mathbf{0} + \begin{pmatrix} \hat{s}_{14}^* & \hat{s}_{15}^* \\ \hat{s}_{24}^* & \hat{s}_{25}^* \\ \hat{s}_{34}^* & \hat{s}_{35}^* \end{pmatrix} + \mathcal{O}(s_{ij}^3). \quad (16)$$

It has been stressed in Ref. [20] that the charged-current interactions of light and heavy Majorana neutrinos are correlated, which is obvious from Eq. (15) and Eq. (16). Furthermore, the extra CP-violating phases come into the non-unitary neutrino matrix V via the matrix A . Thus novel CP-violating effects in the medium-baseline $\nu_\mu \rightarrow \nu_\tau$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau$ oscillations may show up and provide a promising signature of the unitarity violation of V , which could be measured at a neutrino factory [20, 22].

III. COLLIDER SIGNALS OF HEAVY MAJORANA NEUTRINOS

As pointed out in Ref. [5], the generation of neutrino masses and the collider signals of heavy Majorana neutrinos essentially decouple in the realistic seesaw model. Therefore, a

phenomenological approach to consider collider signals of heavy Majorana neutrinos is to take the matrix elements $R_{\alpha i}$ and the heavy Majorana neutrino masses M_i (for $i = 1, 2, 3$) as independent parameters, which should be consistent with both low-energy and current collider experiments. This phenomenological approach has been widely used in the literature [12], however, only for the one heavy Majorana neutrino case. We now generalize previous works and include one more heavy Majorana neutrino, which should be present in the realistic type-I seesaw model [15].

Given the charged-current interactions in Eq. (5), the relevant process reads

$$q(p_1) + \bar{q}'(p_2) \rightarrow l_\alpha^\pm(p_3) N_i(p) \rightarrow l_\alpha^\pm(p_3) + l_\beta^\pm(p_4) + q_f(p_5) + \bar{q}_f'(p_6) , \quad (17)$$

where $\alpha, \beta = e, \mu, \tau$, $i = 1, 2, 3$ and p_1, p_2 etc. represent the four-momentum of the corresponding particles. Heavy Majorana neutrinos can be produced on-shell in this channel, thus it is safe to neglect the contributions from the t -channel diagrams. For simplicity, we consider the typical example with two heavy Majorana neutrinos N_1 and N_2 , while the general situation with more heavy Majorana neutrinos can be analyzed in a similar way. The squared matrix elements for the process in Eq. (17) can be obtained as follows

$$\begin{aligned} |\overline{\mathcal{M}}_N|^2 = & g^8 (2 - \delta_{\alpha\beta}) |D_W(\hat{s}) D_W(q^2)|^2 (p_2 \cdot p_6) \left\{ M_1^2 |R_{\alpha 1} R_{\beta 1}|^2 \mathcal{F}_1 \right. \\ & \left. + M_2^2 |R_{\alpha 2} R_{\beta 2}|^2 \mathcal{F}_2 + M_1 M_2 |R_{\alpha 1} R_{\alpha 2} R_{\beta 1} R_{\beta 2}| \operatorname{Re} [\mathcal{G}^* e^{i\delta}] \right\} , \end{aligned} \quad (18)$$

where $\hat{s} \equiv (p_1 + p_2)^2$, $q \equiv p_5 + p_6$, $\delta \equiv (\delta_{\alpha 1} - \delta_{\alpha 2}) + (\delta_{\beta 1} - \delta_{\beta 2})$, and $R_{\alpha i}$ ($\alpha = e, \mu, \tau$ and $i = 1, 2$) are the mixing matrix elements as indicated in Eq. (16). The explicit expressions of relevant functions D_W , \mathcal{F}_i and \mathcal{G} are collected in Appendix A.

At the hadron collider, the total cross section for the process in Eq. (17) can be expressed as follows

$$\sigma = \sum_{a,b} \int dx_1 dx_2 F_{a/p}(x_1, Q^2) \cdot F_{b/p}(x_2, Q^2) \cdot \hat{\sigma}(ab \rightarrow l_\alpha^\pm l_\beta^\pm q_f \bar{q}_f') , \quad (19)$$

where $F_{a,b/p}$ denote the parton distribution functions for the proton, $x_{1,2}$ the energy fractions of the partons a and b , Q the factorization scale, and $\hat{\sigma}$ the partonic cross section. In our calculations, we consider the reactions at the LHC ($\sqrt{S} = 14$ TeV) and set $|R_{ei}|^2 = 1.25 \times 10^{-7}$, $|R_{\mu i}|^2 = |R_{\tau i}|^2 = 5 \times 10^{-3}$ (for $i = 1, 2$) as well as $\delta = 0$. The factorization scale is taken to be $Q^2 = \hat{s}$. Since the detection of charged leptons μ^\pm is most efficient at the LHC, it is reasonable to explore the $pp \rightarrow \mu^\pm \mu^\pm jj$ processes.

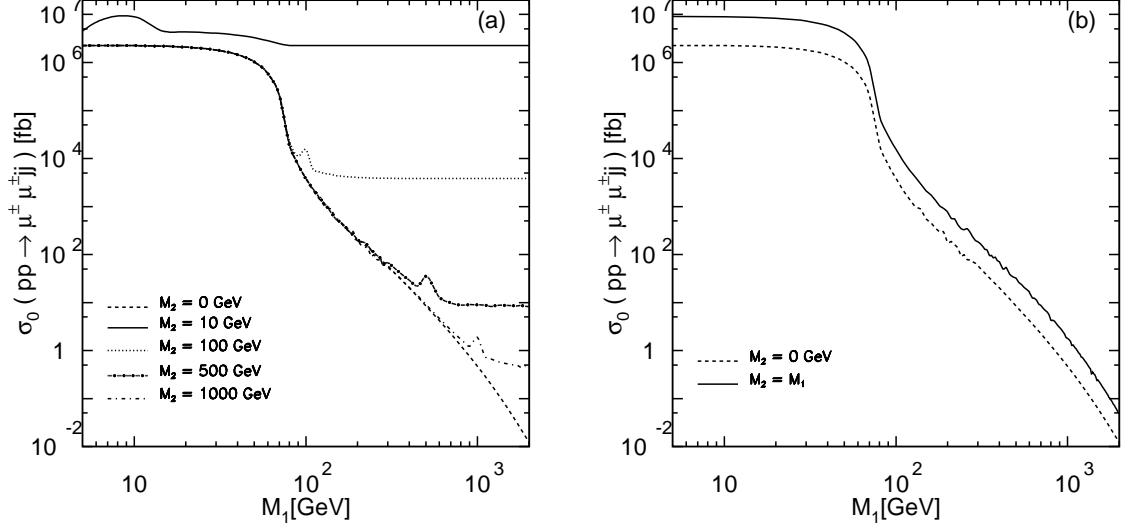


FIG. 1: (a) The cross sections for the same-sign dilepton events $pp \rightarrow \mu^\pm \mu^\pm jj$ with $M_2 = 10$ GeV (solid line), 100 GeV (dotted line), 500 GeV (dotted-solid line) and 1 TeV (dotted-dashed line) versus M_1 varying from 5 GeV to 2 TeV; (b) The solid line corresponds to the case of $M_1 = M_2$. In both panels, the dashed line represents for the reduced cross section with only one heavy Majorana neutrino N_1 .

If only one heavy Majorana neutrino is taken into account, the corresponding cross section for the processes can be decomposed as

$$\sigma(pp \rightarrow l_\alpha^\pm l_\beta^\pm jj) \approx (2 - \delta_{\alpha\beta}) S_{\alpha\beta} \sigma_0, \quad (20)$$

with $S_{\alpha\beta} \equiv |R_{\alpha 1} R_{\beta 1}|^2 / \sum_\gamma |R_{\gamma 1}|^2$ and σ_0 the reduced cross section. Our results for σ_0 with only one Majorana neutrino roughly agree with those in Ref. [12]. For comparison, the same definition for σ_0 is adopted to calculate the reduced cross section in the case with N_1 and N_2 . In FIG. 1(a), we fix the masses $M_2 = 10$ GeV, 100 GeV, 500 GeV and 1 TeV, and change M_1 continuously from 5 GeV to 2 TeV. The resonant enhancement appears where the heavy Majorana neutrino masses are equal $M_2 = M_1$. In the region with $M_1 \gg M_2$, the cross section receives the dominant contribution from N_2 and thus is almost independent of the mass M_1 . We also investigate the interesting case with two degenerate heavy Majorana neutrinos, i.e. $M_2 = M_1$. In FIG. 1(b), it is obvious that the reduced cross section in the degenerate case (solid line) is precisely four times of that with only one heavy Majorana neutrino (dashed line) for the phase difference $\delta = 0$.

The key feature of our signal events is the effective reconstruction of the two heavy

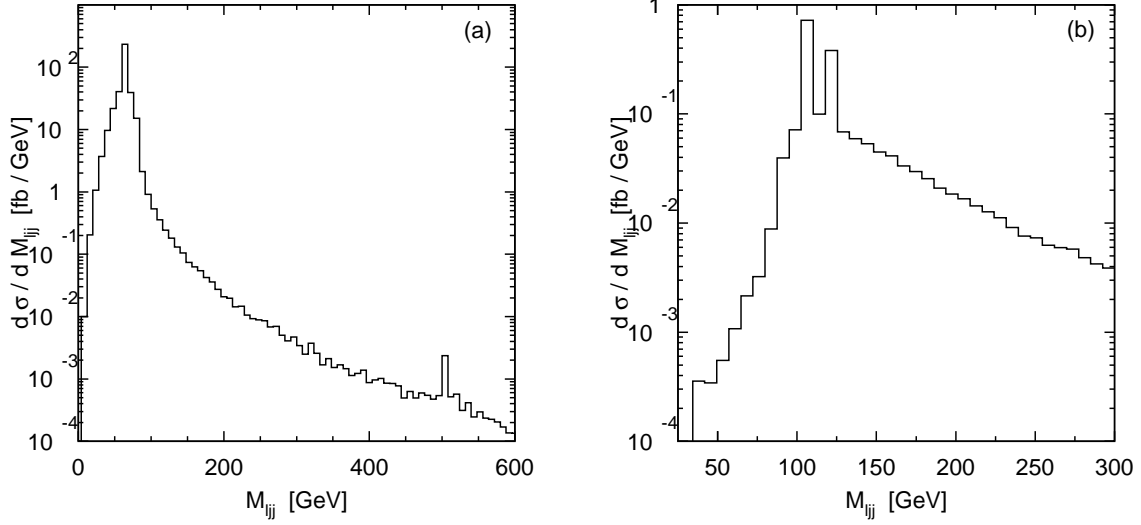


FIG. 2: The invariant mass distribution of charged leptons and jets with (a) $M_1 = 60$ GeV, $M_2 = 500$ GeV and (b) $M_1 = 100$ GeV, $M_2 = 115$ GeV.

Majorana neutrino masses from the final state charged leptons (l_1 and l_2) and jets. Since the final leptons are indistinguishable, it is helpful to define the differential distribution $d\sigma/dM_{ljj} \equiv (d\sigma/dM_{l_1jj} + d\sigma/dM_{l_2jj})/2$, where the invariant masses M_{l_ijk} (for $i = 1, 2$) are constructed from the momenta of related charged-leptons and those of the two jets. In FIG. 2, we show the invariant mass distributions in two different cases: (a) $M_1 = 60$ GeV and $M_2 = 500$ GeV; (b) $M_1 = 100$ GeV and $M_2 = 115$ GeV. It seems from the distribution shape and peak positions that our approach to the reconstruction of heavy Majorana neutrino masses is effective.

To distinguish between the same-sign dilepton signals from heavy Majorana neutrinos and those from the doubly-charged Higgs bosons [10], we compute the differential distribution $d\sigma/d\cos\theta_{\mu\mu}$ and $d\sigma/dM_{\mu\mu}$ of the process in Eq. (17), where $\theta_{\mu\mu}$ is the angle between the final two leptons and $M_{\mu\mu}$ the invariant mass of them. The corresponding results are depicted in FIG. 3(a) and FIG. 3(b) with $M_1(M_2) = 60$ (500) GeV and $M_1(M_2) = 100$ (115) GeV. In FIG. 3(a), it is shown that the differential cross section decreases with increasing $\cos\theta_{\mu\mu}$. While for doubly-charged Higgs bosons, the same-sign dileptons are from the decays of a single scalar particle $H^{\pm\pm} \rightarrow \mu^\pm\mu^\pm$ [23, 24, 25, 26], the corresponding differential cross section is independent of the $\theta_{\mu\mu}$ angle. The scalar particle decay processes also guarantee a peak around the $H^{\pm\pm}$ mass (> 136 GeV [27]) in the invariant mass reconstruction of $M_{\mu\mu}$, while for heavy Majorana neutrinos there is no such signal as shown in FIG. 3(b). These two

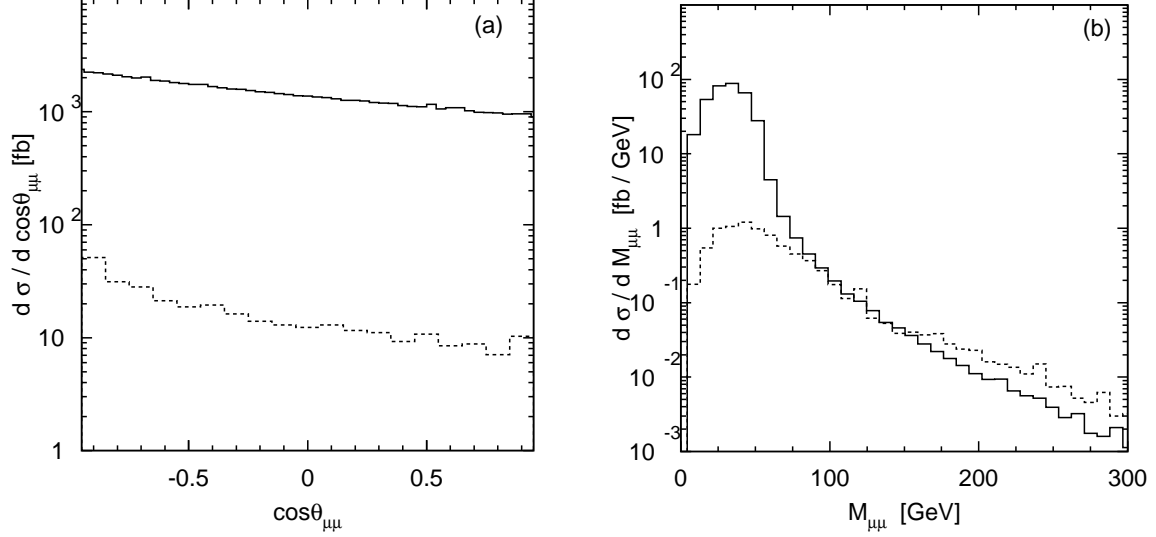


FIG. 3: (a) The angular distribution and (b) the invariant mass distribution of the same-sign dilepton pair with $M_1 = 60$ GeV, $M_2 = 500$ GeV (solid line) and $M_1 = 100$ GeV, $M_2 = 115$ GeV (dashed line).

distributions can serve as an excellent discriminator between the same-sign dilepton signals from heavy Majorana neutrinos and those from the doubly-charged Higgs bosons.

Now we turn to a brief discussion of the SM background and the kinematic cuts used to reduce it. A salient feature of the process in Eq. (17) is the same-sign dilepton with no missing energy in the final states. However, due to the uncertainties in the measurement of the jet energy and electromagnetic energy of charged leptons, the missing transverse energy \cancel{E}_T may appear. To simulate the detector effects on the energy-momentum measurements, we smear the charged-lepton (i.e., electron and muon) and jet energies with a Gaussian distribution as follows

$$\frac{\Delta E}{E} = \frac{a}{\sqrt{E/\text{GeV}}} \oplus b, \quad (21)$$

where $a_l = 5\%$, $b_l = 0.55\%$ for charged leptons and $a_j = 100\%$, $b_j = 5\%$ for jets. The smearing simulation shows that \cancel{E}_T cannot be neglected in the case of $M_1 = 60$ GeV, $M_2 = 500$ GeV. Therefore, we demand that there is no significant missing transverse energy

$$\cancel{E}_T < 25 \text{ GeV}. \quad (22)$$

Furthermore, we adopt the following basic cuts on charged-leptons and jets

- $p_T^l > 10$ GeV and $|\eta^l| < 2.5$;

- $p_T^j > 20$ GeV and $|\eta^j| < 2.5$;
- $\Delta R_{lj} > 0.4$,

where $p_T^{l,j}$ stands respectively for the transverse momentum of the charged lepton and jet, $\eta^{l,j}$ the pseudo-rapidity, and $\Delta R_{lj} = \sqrt{\Delta\eta^2 + \Delta\phi^2}$ the minimal isolation between any two of the final state leptons and jets. At the LHC, the SM contribution to the like-sign dilepton events is pretty small. The leading background comes from the top-quark pair production and its cascade decays via the following chain

$$t \rightarrow W^+ b \rightarrow l_\alpha^+ \nu_\alpha b, \quad \bar{t} \rightarrow W^- \bar{b} \rightarrow W^- \bar{c} \nu_\beta l_\beta^+, \quad (23)$$

The signal and background cross sections, as well as the efficiency of cuts, are given in TABLE I. We see that the background events from $t\bar{t}$ is essentially eliminated by the selective cut of missing transverse energy. In addition, there are two other SM background processes coming from like-sign W boson production. First, the triple gauge-boson production

$$pp \rightarrow W^\pm W^\pm W^\mp \rightarrow l^\pm l^\pm \nu \nu jj, \quad (24)$$

leads to the irreducible background with two like-sign leptons plus jets. Second, the same final states can be produced via the process

$$pp \rightarrow W^\pm W^\pm jj \rightarrow l^\pm l^\pm \nu \nu jj, \quad (25)$$

where the two jets may come either from QCD scattering or from the gauge-boson fusion processes. In our calculations of the background cross sections, we adopt the same couplings and conventions in [28]. After imposing the cuts, we have found that these backgrounds are extremely small, which have been listed in the last two columns in TABLE I.

Our numerical results obtained by inputting some typical values make clear the main features of the collider signals for more than one heavy Majorana neutrinos at the LHC. A systematic analysis of the parameter space is desirable and can be done in a similar way. It is worthwhile to note that a detailed study of the couplings $R_{\alpha i}$ has been performed in Ref. [11], where $R_{\alpha i}$ are reconstructed from low-energy neutrino mixing parameters and heavy Majorana neutrino masses. It has been found that the collider signals are closely correlated with the mass hierarchies of light neutrinos, and also the mass spectra of heavy Majorana neutrinos [11]. From the above discussions, we can conclude that two heavy Majorana

TABLE I: The signal and background cross sections of $pp \rightarrow \mu^\pm \mu^\pm jj$ at the LHC. In the calculation of the signal cross section, we have taken $M_1 = 60$ GeV, $M_2 = 500$ GeV and $|R_{\mu i}|^2 = |R_{\tau i}|^2 = 5 \times 10^{-3} \gg |R_{ei}|^2$ (for $i = 1, 2$) for illustration.

	Signal		$t\bar{t}$		$W^\pm W^\pm W^\mp$		$W^\pm W^\pm jj$	
	$\sigma(\text{fb})$	eff.	$\sigma(\text{fb})$	eff.	$\sigma(\text{fb})$	eff.	$\sigma(\text{fb})$	eff.
Basic cuts	377.7	—	32.0	—	0.23	—	2.04	—
+ \cancel{E}_T cut	243.4	64.4%	4.98	15.6%	0.017	7.39%	0.18	8.82%

neutrinos may induce significant and constructive interference in the total cross section of the same-sign dilepton signals. Moreover, the angular correlation between the final charged leptons and the invariant mass reconstruction from them can provide important information, which may be used to distinguish the LNV signals induced by heavy Majorana neutrinos from those by the doubly-charged Higgs bosons.

IV. CONCLUDING REMARKS

Neutrino oscillation experiments have provided robust evidence that neutrinos are massive. To explain tiny neutrino masses, one should go beyond the SM. Therefore, at the high energy frontier to be explored by the LHC, we also hope to gain some hints on or even to pin down the mechanism of neutrino mass generation. To be specific, we can test the popular seesaw models of neutrino masses at the LHC.

In the canonical seesaw model, the structural cancellation condition $M_D M_R^{-1} M_D^T = \mathbf{0}$ is required to guarantee that the charged-current interactions of heavy Majorana neutrinos are significant, while their masses can be as low as several hundred GeV. Thus the heavy Majorana neutrinos can be discovered at the LHC via the same-sign dilepton signals $pp \rightarrow l_\alpha^\pm l_\beta^\pm jj$ (for $\alpha, \beta = e, \mu, \tau$). Starting from the seesaw formula $M_\nu = M_L - M_D M_R^{-1} M_D^T$ and examining the interplay between the two terms on the right-hand side, we have classified the testable type-(I+II) seesaw model, where both heavy Majorana neutrinos and a triplet scalar are introduced. An intriguing type-(I+II) seesaw model with $M_D M_R^{-1} M_D^T = \mathbf{0}$ and then $M_\nu = M_L$, which is achieved by the discrete $A_4 \times Z_2$ symmetry, has been discussed in some detail. It has been found that (a) the non-unitary neutrino mixing matrix is of the

tri-bimaximal pattern in the leading-order approximation; (b) the heavy Majorana neutrinos are degenerate in mass, so the light neutrino masses are rather stable against the radiative corrections. This scenario is a typical example of the realistic type-(I+II) seesaw model with more than one heavy Majorana neutrinos [29].

Furthermore, we have calculated the cross section of the same-sign dilepton signals $pp \rightarrow l_\alpha^\pm N_i \rightarrow l_\alpha^\pm l_\beta^\pm jj$ (for $i = 1, 2$ and $\alpha, \beta = e, \mu, \tau$) in the minimal type-(I+II) seesaw model. The angular distribution as well as invariant mass distribution of final charged leptons can be used to discriminate the signatures induced by heavy Majorana neutrinos from those by the doubly-charged Higgs bosons. Making use of some kinematic cuts, we have demonstrated that the SM backgrounds can be rendered to be extremely small. It is worthwhile to stress that the constructive interference of the contributions from two heavy Majorana neutrinos may enhance the total signal cross section by a factor up to four. In addition, we put forward an efficient method to reconstruct the masses of heavy Majorana neutrinos. These distinct features of our scenario may show up in the forthcoming CERN LHC, so we hope that the LHC will shed some light on the dynamics of neutrino mass generation in the near future.

APPENDIX A: THE CALCULATION OF CROSS SECTION

In this appendix, we show some details of the calculation of the total cross section for the processes $pp \rightarrow l_\alpha^\pm l_\beta^\pm jj$, which are induced by two heavy Majorana neutrinos N_1 and N_2 . First, we write down the Feynman amplitude for the partonic process in Eq. (17), and the squared matrix element is given in Eq. (18). The relevant functions quoted therein can be cast into a compact form by defining a scalar function

$$D_X(p^2) = \frac{1}{p^2 - M_X^2 + iM_X\Gamma_X} \quad (\text{A1})$$

for the unstable particle X with mass M_X and total decay width Γ_X . For instance, the function $D_W(p^2)$ for the charged gauge bosons W^\pm can be obtained by inputting $M_W = 80.398$ GeV and $\Gamma_W = 2.141$ GeV [30]. Likewise for the heavy Majorana neutrinos N_1 and N_2 . Therefore, the functions \mathcal{F}_i (for $i = 1, 2$) and \mathcal{G} are given by

$$\begin{aligned} \mathcal{F}_i = & \text{Re} [D_i(k_3^2)D_i^*(k_4^2)] (p_1 \cdot p_5) (p_3 \cdot p_4) + \text{Im} [D_i(k_3^2)D_i^*(k_4^2)] \varepsilon_{\mu\nu\lambda\delta} p_1^\mu p_3^\nu p_4^\lambda p_5^\delta \\ & + \{ |D_i(k_4^2)|^2 - \text{Re} [D_i(k_3^2)D_i^*(k_4^2)] \} (p_1 \cdot p_4)(p_3 \cdot p_5) \\ & + \{ |D_i(k_3^2)|^2 - \text{Re} [D_i(k_3^2)D_i^*(k_4^2)] \} (p_1 \cdot p_3)(p_4 \cdot p_5) \end{aligned} \quad (\text{A2})$$

and

$$\begin{aligned}
\mathcal{G} = & \left[2D_1(k_3^2)D_2^*(k_3^2) - D_1(k_4^2)D_2^*(k_3^2) - D_1(k_3^2)D_2^*(k_4^2) \right] (p_1 \cdot p_3)(p_4 \cdot p_5) \\
& + \left[2D_1(k_4^2)D_2^*(k_4^2) - D_1(k_4^2)D_2^*(k_3^2) - D_1(k_3^2)D_2^*(k_4^2) \right] (p_1 \cdot p_4)(p_3 \cdot p_5) \\
& + \left[D_1(k_3^2)D_2^*(k_4^2) + D_1(k_4^2)D_2^*(k_3^2) \right] (p_1 \cdot p_5)(p_3 \cdot p_4) \\
& - \left[D_1(k_3^2)D_2^*(k_4^2) - D_1(k_4^2)D_2^*(k_3^2) \right] i\varepsilon_{\mu\nu\lambda\delta} p_1^\mu p_3^\nu p_4^\lambda p_5^\delta
\end{aligned} \tag{A3}$$

with $k_3 \equiv p_1 + p_2 - p_3$ and $k_4 \equiv p_1 + p_2 - p_4$. Starting with Eq. (18), we can calculate the cross section at the parton level, then the total cross section by using Eq. (19) and also the parton distribution functions CTEQ6L1 [31].

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